**Solution of Week 5.**

**Problem1.**

**There are N risky assets with returns .**

**The expected return and the variance-covariance are denoted by**

**Let be the weights of the risky assets in the portfolio.**

**Using the Lagrange Multiplier Methods, Find the optimal weights of portfolio under given the target expected return**

Let’s find the optimal portfolio with given expected return under the constraint .

For , we define the subject function that

Then, the solution satisfies the followings:

By inserting in the above constraints,

Therefore, the optimal weights of the portfolio are

, , ,

참고:

행렬 편미분 공식<https://m.blog.naver.com/enewltlr/220918689039>

**Problem 2.**

**There are N risky assets with returns and the risk-free asset with risk-free rate .**

**The expected return and the variance-covariance are denoted by**

**Let be the weights of the risky assets in the portfolio. The weight of the risk-free asset is**

**Solve the following sub-problems.**

**1) Using the Lagrange Multiplier Methods, Find the optimal weights of portfolio under given the target expected return**

Let’s find the optimal portfolio with given expected return under the constraint .

For , we define the subject function that

Then, the solution satisfies the followings:

By inserting in the above constraint,

Therefore, the optimal weights under the given expected return are

**2) Find the weight vector of the Tangency Portfolio.**

Let be the weight of the tangency portfolio. Then,

The tangency portfolio exists on the Efficient frontier, so it consists of only risky assets. Therefore, .

we have which gives .

**3) Find the variance of the optimal portfolio.**

We have obtained the optimal portfolio with expected return as follows:

The variance of optimal portfolio is

**4) Use the risk-aversion parameter to indicate the weight of the optimal portfolio. (Definition of risk aversion parameter )**

The risk aversion parameter is

Therefore, the weights of optimal portfolio with risk aversion parameter are

**5) Show that under a given constraint, all optimal portfolios have the same Sharpe Ratio and that all optimal portfolios are above the CAL of the Tangency portfolio.**

Note that

Then, for any target expected return the optimal weights of portfolio are

The definition of Sharpe Ratio (slope of the risk-return plane) is

By the result of sub-problem 3

Because the ratio of risk to return for any optimal portfolio is constant,

Therefore, the Capital Asset Line (CAL) of the tangency portfolio is

**Problem 3.**

생략.

**Problem 4.**

**1) Follow the proof on page 184 of the textbook to induce the formula of CAPM.**

교과서 참조.

**2) Derive the CAPM formula for by using Equation (6.9) in Chapter 6 of the Text book.**

**Hint: Note that Apply equation (6.9) both to asset k and to the market itself.**

Start with Markowitz Mean-Variance optimization,

The optimal portfolio is the market portfolio by the assumption of CAPM. So,

The risk aversion parameter is

Therefore,

**Problem 5. (Optional)**

**Is the beta of securities a constant? Write down your inference.**

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